Signal processing for hemispherical measurement data

Markus Müller-Trapet* and Michael Vorländer

*Corresponding author's address: Institute of Technical Acoustics, RWTH Aachen University, Neustrasse 50, Aachen, 52066, NRW, Germany, mmt@akustik.rwth-aachen.de

To realistically model the sound propagation in rooms, a detailed knowledge of the reflection properties of the surrounding surfaces is required. In this context, the reflection properties include both the sound absorption as well as scattering. In order to be able to measure the angle-dependent reflection properties of surfaces in-situ, a hemispherical microphone array was recently designed and built. For a reduction of the required hardware an efficient, rotationally symmetric sampling was chosen, so that 24 microphones on two concentric semicircles are employed to measure a total of over 2300 positions on a hemisphere in 20 minutes. This contribution will give an overview over the required signal processing steps to process the measurement data from such a microphone array. Special emphasis will be placed on the determination of the microphone positions and the special case of data available on a hemispherical surface. Preliminary results will be presented.

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INTRODUCTION

The measurement of reflection characteristics is of great importance for the generation of material data for room acoustics simulations. If high-quality simulations are desired, the reflection of waves at the room boundaries has to be modeled as exactly as possible. Concerning the reflection factor this has been already shown by several authors [1, 2, 3]. Reflections at boundaries are, however, not only dependent on the absorbing characteristics of the surface material, they also depend on the geometrical shape, which causes the waves to be scattered in non-specular directions for certain frequencies.

Measuring the absorbing as well as the scattering properties of surfaces is possible using standardized laboratory procedures [4, 5, 6]. One shortcoming of these laboratory measurement standards is that they only acquire the results for either normal-incidence (ISO10534-2) or random-incidence (ISO354 and ISO17497-1). A recent standard to measure the diffusion coefficient [7] under laboratory conditions does measure the parameter depending on the direction of the incident wave but not a lot of data is available yet from these measurements.

Another problem with laboratory measurements is that they demand for the sample to be cut to a specific size, thus changing the material from its built-in condition. Sometimes this is even unfeasible especially once the material is installed in the desired location.

In order to overcome the restrictions and drawbacks of laboratory measurement techniques, an array setup for the in-situ measurement of surface properties was designed, built and tested. The mechanical setup is transportable and adjustable in height so that it can be placed above a test sample on the ground. The setup uses 24 microphones distributed over two semicircles in combination with a step motor to measure the sound pressure distribution over a complete hemisphere. This approach dramatically reduces the necessary hardware while being able to measure at a set of 2304 discrete measurement positions. The principle of this sequential array is depicted in Figure 1.

![Figure 1: Principle of the sequential microphone array. The dots represent the microphone positions.](image)

Once the sound pressure distribution on the hemi-spherical shell is known, the data can be used – after adequate post-processing – to derive parameters like the complex reflection factor or the scattering coefficient, each depending on the angle of sound incidence.

This contribution will present some of the necessary steps to process data from such a measurement. The sound pressure distribution will be analyzed by transforming the data into the Spherical Wave Spectrum. For this task it is first important to determine the actual microphone positions as they will almost always differ from the ideal positions and positional errors will affect the spatial transformation.
MEASUREMENT SETUP

Measurements were carried out in the semi-anechoic chamber of the Institute of Technical Acoustics (ITA) in Aachen. The array was held by a mechanical construction made of aluminum bars, which can be disassembled for transportation and which allows for an exact positioning above the test sample. Two test cases were investigated for the results presented in this paper: a setup above the perfectly reflecting floor and one above a porous absorber made of pu-foam. Figure 2(a) shows the setup in the semi-anechoic chamber above a sample of pu-foam. In Figure 2(b) the microphone mounts on one of the two arcs are depicted. The source – on a stand in the foreground in Figure 2(a) – used for the measurements is a loudspeaker driver in a custom-built spherical enclosure, which results in minimal edge diffraction and hence a very smooth frequency response without larger dips up to roughly 10 kHz (Figure 3).

![Array setup](image1)

(a) Array setup

![Close-up of the microphone mounts](image2)

(b) Close-up of the microphone mounts

**Figure 2:** Measurement setup for the sequential array in the semi-anechoic chamber. Shown is the setup above a porous absorber.

![Loudspeaker](image3)

(a) Loudspeaker

![Frequency Response](image4)

(b) Frequency Response

**Figure 3:** Measurement loudspeaker used for all measurements together with the frequency response.

The 24 physical microphones of the sequential array are configured in a Gaussian quadrature
sampling scheme [8] at two radii of 0.512 m and 0.527 m (blue and green arc in Figure 1, respectively). The array has to be turned 96 times in order to reach the starting position again, which means that a single turn corresponds to 3.75 degree. A complete measurement thus results in \(24 \cdot 96 = 2304\) positions and takes about 20 minutes.

Impulse responses with a length of \(2^{18}\) samples were recorded at a sampling rate of 48 kHz for each turn of the arcs by using exponential sweeps, allowing for a removal of slight non-linearities caused by the loudspeaker and a reduction of background noise effects [9]. The system latency was also measured and compensated for, so that the measured arrival times correspond to the acoustic propagation time.

**DATA PROCESSING**

After assembling and sorting all impulse responses, the data was windowed to reduce the effects of non-linearities and background noise. After that, each impulse response was oversampled tenfold and the arrival time of the impulse was determined and saved for the later analysis. The further processing steps, which include an optimization of the microphone locations as well as the subsequent transform into the Spherical Wave Spectrum will be described in the following paragraphs.

**Position Estimation**

A crucial part of measurements with microphone arrays is an exact knowledge of the microphone positions as array-processing techniques are very sensitive to deviations from the ideal positions [10].

In this context, several sources of error can be established, which are a Cartesian displacement of the array (\(\Delta \mathbf{M}\)) and the loudspeaker (\(\Delta \mathbf{L}\)) from their intended positions and individual Cartesian positioning errors of the 24 microphones on the arcs (\(\Delta \mathbf{M}_i\)). All of these sources of error have been taken into account in an optimization process to determine and correct for these errors.

Let \(\mathbf{L}\) be the Cartesian coordinate vector of the loudspeaker position, \(\mathbf{M}_i\) the microphone coordinates of the microphone with index \(i\) and \(\tau_i\) the arrival time of the impulse response at microphone \(i\), then the deviation between theoretical and actual distances between loudspeaker and the microphone with index \(i\) can be expressed as

\[
\Delta d_i = \|\mathbf{L} - \mathbf{M}_i\| - \tau_i \cdot c, \tag{1}
\]

where \(c\) is the speed of sound and \(\|\cdots\|\) is the vector norm. If the aforementioned sources of error are included, Equation 1 can be altered with the goal of determining the errors in an optimization process:

\[
\Delta d_{i, \text{opt}} = \|\mathbf{L} + \Delta \mathbf{L} - (\mathbf{M}_i + \Delta \mathbf{M} + \Delta \mathbf{M}_i)\| - \tau_i \cdot c. \tag{2}
\]

The correction terms in this equation can be obtained by finding the minimum of Equation 2 across all measurement positions through a least-squares optimization, where it has to be noted that the system is overdetermined, as the errors in the microphone locations are not independent. An offset in each of the 24 physical microphones will translate during the rotation of that microphone to achieve the virtual microphone positions at the same elevation. In conclusion, 2304 known values of \(\Delta d_{i, \text{opt}}\) exist for a total of \((2 + 24) \cdot 3 = 78\) unknowns, which account for the Cartesian displacement of the loudspeaker, the array and the offset for each of the 24 microphones in all Cartesian directions.
Spherical Wave Spectrum

For measurements on spherical shells, the transformation of the data from the Cartesian into the Spherical Harmonics (SH) domain is very suitable. The SH transform is the two-dimensional spatial Fourier transform, in analogy to the Fourier transform between time and frequency data. Most of the formulas used in this paper are based on the work by Williams [11], though due to a different sign convention they may vary slightly.

The pair of transforms of a sound pressure distribution on a spherical shell of radius \( r \) from the spatial domain into the SH domain and vice versa is given by:

\[
p_{nm}(k, r) = \frac{2\pi}{\int_0^\pi \int_0^{2\pi} p(k, r, \theta, \phi) Y_n^m(\theta, \phi) \sin(\theta) d\theta d\phi,}
\]

\[
p(k, r, \theta, \phi) = \sum_{n=-\infty}^{\infty} \sum_{m=-n}^{n} p_{nm}(k, r) Y_n^m(\theta, \phi),
\]

where \( k = \frac{\omega}{c} \) is the wavenumber, \( Y_n^m \) is the spherical harmonic base function of order \( n \) and degree \( m \) and \( p_{nm} \) are the spherical harmonics coefficients. The operator \((*)^*\) denotes the complex conjugate. In practice, the order \( n \) cannot be infinitely high, and hence the system of equations has to be truncated at a certain order \( N_{\text{max}} \), which depends on the spatial sampling and the frequency. For the data presented here, the maximum order was \( N_{\text{max}} = 47 \), which permitted an SH transform without aliasing at least up to 5 kHz. The truncation gives the possibility to formulate the transforms in Equations 4 as a matrix-vector product by stacking the base functions as well as the coefficients into the matrix \( Y \) and the vector \( p_{nm} \):

\[
p = Yp_{nm}.
\]

The task of determining the unknown coefficient vector \( p_{nm} \) from measured data according to Equation 3 is then a problem of matrix inversion. For the data presented here the pseudo-inverse was employed.

Point Source and Extrapolation

Modeling the sound propagation from a point source with constant unit amplitude at location \((R, \theta_0, \phi_0)\) towards a spherical array of radius \( r \) in the SH domain is possible using the expansion of the Green's function into spherical harmonics coefficients [12]

\[
p_{nm}(k, r) = -i k \cdot j_n(kr) h_n^{(2)}(kR) Y_n^m(\theta_0, \phi_0),
\]

where \( j_n \) is the spherical Bessel function of order \( n \) and \( h_n^{(2)} \) is the spherical Hankel function of the second kind. It can be observed that the dependency of the coefficients on the observation radius \( r \) lies solely in the Bessel function term. This means that once the coefficients \( p_{nm}(k, r) \) are known for one distance, they can be calculated for any other distance \( r_0 < R \):

\[
p_{nm}(k, r_0) = p_{nm}(k, r) \frac{j_n(kr_0)}{j_n(kr)}.
\]

This is known as the extrapolation problem and has been successfully applied in other situations [13].

In this contribution, the radii of the different microphone positions were determined through the optimization process described before. In this case, when there are multiple values for the radius \( r \), determining the spherical harmonics coefficients can be performed by combining the
base functions $Y_n^m$ and the Bessel function term into one matrix, which can then be inverted. This processing was performed on the measurement data and the result was later re-expanded to one common radius $r_0 = 0.51 \, \text{m}$.

Symmetry of Spherical Harmonics

Originally, the spherical harmonic base functions are defined over the complete sphere. For the hemispherical array presented in this contribution, the base functions or the data have to be adapted. The simplest way would be to mirror the hemispherical data with respect to the z-axis in order to obtain data on a complete sphere. However, this would create redundant data without additional information and so a better approach is to directly exploit the symmetry of the base functions.

Some of the base functions $Y_n^m$ exhibit a symmetry with respect to the z-axis, which means that by choosing only the upper hemisphere no information is lost. By picking only the symmetrical base functions where $n + m$ is an even number, a new base for the transform is created which can then directly be applied to the data available on a hemisphere [14].

RESULTS

Figure 4 shows the result of the optimization process for the determination of the microphone positions. It can be observed that before the optimization there is an offset of roughly 2.5 cm and a large positional error of up to 6 cm. After the optimization, which takes less than one minute, the average error is reduced significantly to less than 0.5 cm. Several peaks still appear in the graph, which are related to the microphone indices of two rotational positions of the array. It is unclear yet, where this remaining error comes from, but it is still a lot less than the initial error.

![Graph showing positional error and microphone positions](image)

**Figure 4:** Result of the optimization process for the determination of the actual microphone positions based on the arrival times.

The optimization process is relatively robust as it is overdetermined, which makes it possible to give only rough initial estimates for the loudspeaker and array positions to obtain a correct result. This has been tested with different setups above different samples and the positional errors always converged to the results presented in Figure 4.

Figure 5 shows the effect of the post-processing in the SH domain. Depicted are balloon plots of the complex pressure distribution at different frequencies, where the radius of the balloon
corresponds to modulus and the color corresponds to phase. The figure shows the results for the array above the reflecting floor, where the loudspeaker stood at a radius of 1.8 m to the array center at an elevation angle of roughly 45 degrees. It can be noticed that the transformation and re-expansion correctly preserves the phase and smoothes out minor irregularities in the amplitude.

**FIGURE 5:** Result of the post-processing in the SH domain.
CONCLUSION

In this contribution, a hemi-spherical array setup for the in-situ measurement of surface properties has been presented together with the necessary signal processing. An optimization process for the determination of the microphone positions was outlined and tested, and good results were achieved concerning the positional errors, although two positions still showed uncertainties in the measurement positions.

The theory for the transformation of pressure data on a hemi-spherical shell was presented and applied to the measured data. It could be shown that the range extrapolation worked for the data shown here. The correct reproduction of the phase and amplitude confirms the validity of the transform and its applicability, as especially the phase information is important for any subsequent analysis with regard to the complex reflection factor.

The processed data can now be used in order to determine surface parameters like the complex reflection factor. Here, the extrapolation approach is very convenient, as this means that the evaluation of the pressure above the reflecting surface can be done at arbitrary positions, giving the possibility to evaluate the reflection factor for different angles of incidence.

The fact that the microphones are positioned at two different radii on the arcs can be exploited when employing a technique called scattering nearfield holography [11, Chapter 7.4]. This would make a reference measurement of the loudspeaker obsolete as it could directly be obtained from the array measurement [15].

The presented array setup can be used to gain a better understanding of how sound is reflected off different surfaces.

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REFERENCES


