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4pSP11. Nested sampling-based design of multilayer microperforated panel sound absorbers
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A model-based design approach for microperforated panel absorbers comprised of multiple panel layers is developed. Microperforated panels (MPPs) are becoming increasingly popular as sound absorbers, capable of providing broadband absorption with high absorption coefficients, without the use of traditional porous materials. To increase the bandwidth of the intrinsically peaked narrowband absorption of a single MPP, multiple such panels can be combined into composite sound absorbers. We propose a method based on Bayesian inference to design multilayered MPP absorbers capable of producing a user-specified absorption profile. Using nested sampling to accumulate Bayesian evidence and to implement Occam's razor, the method produces a design requiring the fewest number of MPP layers while meeting the specified design requirements.

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INTRODUCTION

In the context of architectural acoustics, porous materials have traditionally been used to provide acoustic absorption. Microperforated panel absorbers, first proposed by Maa [1], are capable of providing very high acoustic absorption without the use of porous or fibrous materials; however, this absorption is generally concentrated over narrow frequency ranges. By constructing multilayer absorbers, consisting of stacked microperforated panels, the absorption bandwidth may be widened to cover broad frequency ranges. We develop a Bayesian inference-based method for designing such multilayer microperforated panel absorbers, capable of determining the number of layers and the design parameters of each layer necessary to produce a desired absorption result.

NUMERICAL MODELING

Atalla-Sgard Model

Atalla and Sgard proposed a model [2] for the acoustic response of a microperforated panel, treating the panel as an equivalent fluid and modifying the existing Johnson-Allard rigid frame porous model. A microperforated panel is described by three parameters, the panel thickness $t$, the radius of the perforations $r$, and the perforation percentage or porosity $\phi$. The surface impedance of a microperforated panel mounted in front of an air cavity is determined as:

$$Z_S = Z_A + Z_C,$$

where $Z_A$ is the acoustic impedance of the panel itself and $Z_C$ is the acoustic impedance of the cavity backing the panel.

Using the Atalla-Sgard model, the acoustic impedance of a microperforated panel is given as:

$$Z_A = (t + 2\epsilon_e) \left[ (1 + i) \frac{2R_s}{\phi} + \frac{i\omega\rho_0}{\phi} \right],$$

where $\epsilon_e = 0.48r/\sqrt{\pi}$ is a correction length, $R_s = \sqrt{\eta\omega\rho_0/2}$ is the surface resistance, and $\omega$ is the angular frequency at which to calculate the impedance. The air saturating the perforations and surrounding the panel has density $\rho_0$ and dynamic viscosity $\eta$.

For the case of a microperforated panel mounted a distance $L$ in front of a rigid backing, an air cavity of depth $L$ is formed. The acoustic impedance of such a cavity is:

$$Z_C = -iZ_0\cot(\beta L),$$

where $Z_0 = \rho_0 c$ is the characteristic acoustic impedance of the air, $\beta = \omega/c$ is the phase coefficient of the air, and $c$ is the speed of sound. Thus, by substituting Equations (2) and (3) into Equation (1), the surface impedance of a single microperforated panel mounted in front of a rigid wall is determined.

Multilayer Panel Model

Multiple microperforated panels may be stacked to produce a composite absorber, as illustrated in Figure 1. In this arrangement, only the first panel closest to the wall is backed by an air cavity terminated by a rigid backing, thus having a cavity impedance $Z_{C1}$ given by Equation (3).

Each subsequent air cavity is backed by the input impedance of the preceding microperforated panel, rather than a rigid wall. Thus, for cavities 2 through $N$, the normal-incidence input impedance to the $n$-th cavity may be calculated with an impedance transfer approach [3], yielding:

$$Z_{Cn} = Z_0 \frac{Z_{n-1} \cos(\beta L_n) + iZ_0 \sin(\beta L_n)}{Z_0 \cos(\beta L_n) + iZ_{n-1} \sin(\beta L_n)},$$

where $Z_0$ is again the characteristic acoustic impedance of the air, $L_n$ is the depth of the $n$-th air cavity, and $Z_{n-1}$ is the normal-incidence input impedance of the microperforated panel backing the $n$-th cavity.
An iterative procedure is used to compute the overall acoustic properties of a multilayer absorber, beginning with the layer adjacent to the rigid termination. The first cavity impedance is evaluated with Equation (3), and the first panel impedance is evaluated with Equation (2). The input impedance to the first panel, as computed by Equation (1), serves as the backing impedance to the second cavity. Subsequent cavity impedances are calculated with Equation (4). The calculation proceeds until the input impedance to the \( N \)-th (outermost) panel provides the surface impedance \( Z_S \) of the overall absorber.

\[
\begin{align*}
Z_N & \quad Z_{N-1} & Z_2 & \quad Z_1 \\
Z_{C_N} & \quad Z_{C_2} & \quad Z_{C_1} \\
L_N & \quad L_2 & \quad L_1
\end{align*}
\]

**Figure 1:** Multilayer microperforated panel absorber layout. Each microperforated panel layer is backed by an air cavity of depth \( L_n \) with impedance \( Z_{C_n} \). The input impedance of each panel is \( Z_n \).

Given the normal surface impedance for the compound absorber \( Z_S \), the normal-incidence reflection coefficient for the absorber is calculated as:

\[
R = \frac{Z_S - Z_0}{Z_S + Z_0},
\]

(5)

from which the normal-incidence absorption coefficient is calculated as:

\[
\alpha = 1 - |R|^2.
\]

(6)

**Design Framework**

**Bayesian Inference**

Bayesian inference provides a probabilistic framework for scientific reasoning, where probabilities measure a degree of belief. This inference framework is especially suited to solving inverse problems, where a model describes data in terms of parameters of interest. The parameters are difficult or impossible to determine directly, and are thus determined inversely from the available data.

At the heart of the Bayesian inference framework is Bayes’ theorem, which describes how prior beliefs should be updated in the presence of new data. Bayes’ theorem may be stated as:

\[
\underbrace{p(\theta|D,H)}_{\text{posterior \( P \)}} \times \underbrace{p(D|H)}_{\text{evidence \( W \)}} = \underbrace{p(\theta|H)}_{\text{prior \( \pi \)}} \times \underbrace{p(D|\theta,H)}_{\text{likelihood \( L \)}},
\]

(7)

where each of the four probability terms – written in terms of the parameter values of interest \( \theta \), experimental or observational data \( D \), and model \( H \) – serves a different purpose and has a typical name notated below. All probabilities in Equation (7) are also implicitly conditioned on any available background information \( I \); this conditioning has been dropped to simplify the notation. In a typical Bayesian inference computation, the prior and likelihood are assigned as inputs and the evidence and posterior distribution are the resulting outputs.

The prior distribution \( \pi = p(\theta|H) \) encodes any knowledge about the parameter values \( \theta_i \) before the analysis begins. Often the only available knowledge is that the parameter values lie somewhere within a given range. Since these are location parameters, this complete ignorance of value is represented by assigning a flat prior probability distribution, bounded to the applicable value range [4].
In model-based inference, the likelihood function \( L = p(D|\theta,H) \) represents the degree of belief that a given model and parameter values generate a set of observed data. The likelihood function serves to update the prior knowledge after data has been observed. The assignment of likelihood function for inference-based design is covered in the next section.

The evidence \( W = p(D|H) \) is proportional to the degree of belief that a given model could have generated the observed data, regardless of parameter values. In a situation where several models may describe a set of data, the models may be ranked by their respective evidence values. The model with highest evidence should be believed to have generated the data, and thus selected for use in inferring the parameter values.

Finally, given the prior distribution, likelihood function, and evidence value, the posterior distribution \( P = p(\theta|D,H) \) encodes all information about the parameters that has been gained through the Bayesian analysis.

**Likelihood Assignment**

Our design goal is to produce a multilayer microperforated panel absorber that produces a desired acoustic absorption profile. To achieve this goal, we construct a tolerance scheme, such as illustrated in Figure 2. At each frequency of interest, acceptable upper and lower limits for the acoustic absorption coefficient are specified by the designer. As an example, in the context of room acoustics, these limits may be chosen to introduce targeted absorption into a room to reduce excess reverberation.

From this tolerance scheme, we can extract an upper limit boundary \( B_U(\omega_m) \) and a lower limit boundary \( B_L(\omega_m) \) at any angular frequency \( \omega_m \) of interest. For our design problem, we select \( M \) such frequencies. The acoustic absorption coefficient at each frequency, \( \alpha(\omega_m) \) is given by the model for multilayer microperforated panels described in Section 2.2.

In formulating the likelihood function, the data \( D \) consists of the upper and lower limit boundaries, \( B_U \) and \( B_L \). The parameters \( \theta \) consist of the physical parameters used to model a microperforated panel: panel thickness, perforation radius, perforation percentage, and air cavity depth. For an \( N \)-layer absorber, \( N \) sets of these parameters are used for the model \( H \).

The likelihood function is derived from the error between the desired acoustic absorption (specified as the upper and lower limits in the tolerance scheme) and the modeled absorption provided by the multilayer panel.
model. Adapting the filter design work of Chan and Goggans [5] to the multilayer microperforated panel problem at hand, the error at each frequency $\omega_m$ is defined as:

$$E_m = \begin{cases} 
B_L(\omega_m) - a(\omega_m) & \text{for } a(\omega_m) \leq B_L(\omega_m), \\
B_U(\omega_m) - a(\omega_m) & \text{for } a(\omega_m) \geq B_U(\omega_m), \\
0 & \text{otherwise.}
\end{cases}$$  \hspace{1cm} (8)

Following from this error, the likelihood function at each frequency is heuristically assigned a Gaussian probability density function. Since the errors at each frequency are independent, the product rule is applied yielding the overall likelihood:

$$p(D|\theta, H) = \prod_{m=1}^{M} \frac{1}{\sigma_m \sqrt{2\pi}} \cdot \exp\left(\frac{E_m^2}{2\sigma_m^2}\right), \hspace{1cm} (9)$$

where $\sigma_m$ is the standard deviation assigned by the designer at each frequency, which governs how closely the resulting design will agree with the specified tolerance scheme. Smaller values of $\sigma_m$ will result in better agreement between the given specifications and the resulting design.

**NESTED SAMPLING**

Nested sampling [6] is a numerical algorithm for computing the evidence and posterior in a Bayesian inference problem, given the prior and likelihood as inputs. The nested sampling procedure begins with a population of $N$ sample objects, which have been sampled according to the prior distribution. Nested sampling exploits the intimate connection between the likelihood function and the prior mass. When the prior mass $\epsilon$ is at its maximum ($\epsilon = 1$), the likelihood is at its minimum: $L(\epsilon_{\text{max}}) = 0$. Likewise, when the prior mass is minimized ($\epsilon = 0$) the likelihood function is maximized: $L(\epsilon_{\text{min}}) = L_{\text{max}}$ (given that this maximum exists). At intermediate states, $L(\epsilon)$ refers to the likelihood value when an amount $\epsilon$ of prior mass remains at higher likelihood values.

At each step of the iterative procedure, the sample in the population of $N$ having the lowest likelihood value is recorded and discarded. This likelihood creates a constraint, such that the likelihood values of the remaining $N - 1$ samples are greater than that of the discarded sample. One of these survivors is selected at random, duplicated, and evolved to generate a new sample, also obeying the constraint that its likelihood is greater than that of the discarded sample. Thus, after each iteration, a population of $N$ samples exists which are uniformly distributed over the prior mass enclosed within the limiting likelihood value.

Ultimately the goal of nested sampling is to compute the evidence and obtain samples from the posterior distribution. The evidence is given as the product of prior mass and likelihood, accumulated over all likelihood samples throughout the sampling procedure. After each iteration, the prior mass covered by the population of samples has been compressed. This shrinkage of the prior distribution is estimated (as 1 part in $N$ since the samples remain uniformly distributed over the constrained prior) so that the product of remaining prior mass and likelihood may be computed after each step to accumulate evidence. By appropriately weighting these samples, random samples from the posterior distribution are obtained simultaneously.

**RESULTS**

Given the design limits sketched in Figure 2 we set out to produce a multilayer microperforated absorber utilizing the fewest layers and still meeting the design criteria. We considered designs encompassing between 1 and 4 layers, assigning the following prior probability density functions to the parameters for each layer based on typical values covering a wide range of possible microperforated panel designs:

$$p(\text{thickness } t) = \text{Uniform}(0.5\text{mm}, 3\text{mm}), \hspace{1cm} (10)$$
$$p(\text{perforation radius } r) = \text{Uniform}(0.0\text{mm}, 0.5\text{mm}), \hspace{1cm} (11)$$
$$p(\text{porosity } \phi) = \text{Uniform}(0\%, 20\%), \hspace{1cm} (12)$$
$$p(\text{cavity depth } L) = \text{Uniform}(0.5\text{cm}, 5\text{cm}). \hspace{1cm} (13)$$
TABLE 1: Evidence values for a multilayer microperforated panel absorber comprising various numbers of layers. Since it has the highest evidence value, it is seen that a three-layer absorber is the minimal complexity required to satisfy the design constraints.

<table>
<thead>
<tr>
<th># Layers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Evidence (Np)</td>
<td>-849.96 ± 0.604</td>
<td>130.91 ± 0.584</td>
<td>138.95 ± 0.584</td>
<td>133.24 ± 0.600</td>
</tr>
</tbody>
</table>

The model is evaluated and compared to the design requirement at 101 points, evenly spaced from 1 Hz to 5 kHz. To control the degree of compliance between the design specifications and the resulting absorber design, the following standard deviations \( \sigma_i \) are assigned:

\[
\sigma_i = \begin{cases} 
0.2 & 0 \text{Hz} \leq f_i < 750 \text{Hz}, \\
0.01 & 750 \text{Hz} \leq f_i < 2 \text{kHz}, \\
0.05 & 2 \text{kHz} \leq f_i < 4 \text{kHz}, \\
0.2 & 4 \text{kHz} \leq f_i \leq 5 \text{kHz}.
\end{cases}
\]  

(14)

Table 1 lists the evidence values calculated after running the nested sampling calculation for each of the four multilayer configurations. The design corresponding to three layers has the highest evidence value. This indicates that, for the given design constraints, a three-layer panel is the simplest design capable of satisfying the design requirements.

After determining a three layer panel is required to meet the design requirements, the parameters for each layer are extracted from the posterior probability distribution describing the three layer design. By finding the parameter values corresponding to the point of greatest posterior probability density, the three layer microperforated panel design is specified. Figure 3 reproduces the design requirements along with the absorption of the three layer panel design.

**CONCLUSIONS**

We have developed an inference-based system for designing multilayer microperforated absorbers. A desired acoustic absorption response is specified by the designer in terms of upper and lower absorption
boundaries at each frequency of interest. With application of Bayesian inference and nested sampling, the minimum number of layers necessary to achieve the desired absorption are determined alongside the physical design parameters for each layer.

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REFERENCES


